Performance Measures

This note provides a detailed description of the measures used in this study to quantitatively and objectively evaluate the performance of the particle tracking methods.

INTRODUCTION

The problem of performance evaluation of tracking methods occurs in many fields, including computer vision, aerospace applications (radar tracking, navigation, traffic control), and biomedical research. Despite much consideration in the past decades,[1-3] there is as yet no single, well-accepted method to evaluate overall tracking performance. This can be explained by the fact that different application areas may be concerned with different aspects of track estimation and, consequently, may require different performance measures. In particular, measures proposed in other fields are often not applicable to biological particle tracking, where one is faced with a priori unknown and varying numbers of particles, whose identities are to be preserved throughout the image sequence.[4,5]

A key aspect of comparing a set of estimated objects to a set of known but possibly a different number of ground-truth objects, is the pairing of their elements: which element in the former should be compared to which element in the latter? A sensible approach to solving this problem for sets of positions is the use of optimal subpattern assignment.[6,7] This concept has recently also been extended to sets of labeled object tracks.[8] The evaluation of particle tracking methods in the present study was based on the same underlying idea, as described in detail below. Nevertheless, in order to have a complete and intuitive characterization of the performance of the different methods, a set of complementary performance measures was used, rather than a single measure.

TRACK DEFINITION

A track is a temporal series of subsequent spatial positions. The spatial position at a given time point $t \geq 0$ is a vector $\theta(t) = (x(t), y(t), z(t))$, with $x(t)$, $y(t)$, and $z(t)$ the coordinates at this time along the respective axes of the image. In a 3D image sequence, all three coordinates may vary, while in a 2D image sequence, the z coordinate is fixed. A track θ existing from time $t_{\text{init}} \geq 0$ to time $t_{\text{end}} \geq t_{\text{init}}$ is therefore defined as the set $\theta = \{\theta(t), t = t_{\text{init}}, \dots, t_{\text{end}}\}$. Missing positions in the interval $[t_{\text{init}}, t_{\text{end}}]$ are marked as non-matching and are penalized as described below.

DISTANCE BETWEEN TWO TRACKS

For the purpose of measuring the distance between two tracks, the following gated Euclidean distance between two positions $\theta_1(t)$ and $\theta_2(t)$ is defined:

$$\|\theta_1(t) - \theta_2(t)\|_{2,\epsilon} \triangleq \min(\|\theta_1(t) - \theta_2(t)\|_2, \epsilon)$$

with $\|\cdot\|_2$ the standard l_2 norm of R_3, and $\epsilon \in \mathrm{R}_+$ the gate. The rationale behind the use of the gate ϵ is to limit the penalization of tracks that separate. When two tracks are more than ϵ apart at any time t, it is indeed considered that their positions do not match at that time point. In that case, it is irrelevant to measure the actual distance between these positions, and a fixed penalty ϵ is used instead. In the context of this study, the value of ϵ was set to 5 pixels, which, for the imaging parameters simulated in our data, was on the order of the Rayleigh criterion.[9,10] In other words, the required minimum distance between diffraction-limited particles to allow visual separation (Rayleigh), was taken as the maximum tolerable distance for the particle tracking methods.

It may happen that two tracks have different temporal supports. For instance, θ_1 may exist at a given time t, while θ_2 does not. In that case, we consider that the tracks do not match at that time point, and the distance between the tracks is defined to be equal to the penalty ϵ. If neither of the two tracks exist at time t, their distance is defined to be 0. This allows for the following compact formulation of the distance d between any two tracks θ_1 and θ_2:

$$d(\theta_1, \theta_2) \triangleq \sum_{t=0}^{T-1} \|\theta_1(t) - \theta_2(t)\|_{2,\epsilon}$$

where T is the length (the number of frames) of the image sequence.

DISTANCE BETWEEN TWO TRACK SETS

Let $X = \{\theta_k^X, k = 1, \ldots, |X|\}$ be an ordered set of ground-truth tracks, and Y a set of estimated tracks, whose similarity to X needs to be evaluated. Since some tracks in Y may not match a track in X, or vice versa, Y is extended with $|X|$ dummy tracks that are empty. Let $\tilde{Y}$ denote this extended set of estimated tracks. Furthermore, let Ω be the ensemble of ordered sets of tracks that can be obtained by taking $|X|$ elements from $\tilde{Y}$. The distance between any $Z \in \Omega$ and X is then defined as the sum of the distances between the $|X|$ pairs of tracks given by the ordering of the two sets. This allows for the definition of the distance between X and Y as the minimum distance between X and all possible Z:

$$d(X,Y) \triangleq \min_{Z\in\Omega} \sum_{k=1}^{|X|} d\left(\theta_k^X, \theta_k^Z\right)$$

Building the set of tracks $Z^* \in \Omega$ that minimizes the distance to X, involves reordering $\tilde{Y}$ and taking a subset of $|X|$ elements from it. This task can be viewed as a rectangular assignment problem between the tracks in X and $\tilde{Y}$. Because of the additivity and positivity of the cost of track association according to the above definition, this problem can be solved in polynomial time, using the Munkres algorithm.[11]

PERFORMANCE MEASURES

In order to evaluate the performance of any particle tracking method for any data, the output track set Y of the method was scored with respect to the ground-truth track set X of that data using the following measures, based on the optimal pairing (X, Z^*) described above:

1) The measure $\alpha(X,Y) \triangleq 1 - d(X,Y)/d(X,\emptyset)$, where $\emptyset$ denotes the set of $|X|$ dummy tracks. By definition of $\|\cdot\|_{2,\epsilon}$, the lower bound of $d(X,Y)$ is 0, and the upper bound is $d(X,\emptyset)$. Indeed, a pair of tracks (θ_i^X, θ_j^Y) is guaranteed not to be selected by the optimization process if the distance between them is larger than the distance between θ_i^X and a dummy track. The value of measure $\alpha(X,Y)$ therefore lies in the interval $[0,1]$. It takes value 1 if the pairs of tracks in X and Z^* match exactly (the distance between each pair of tracks is 0). It takes value 0 if no valid match could be found, that is if $Z^* = \emptyset$. It scores the best possible pairing of tracks between X and Y, and ignores the tracks in Y that did not make it into Z^*.
2) The measure $\beta(X,Y) \triangleq \left(d(X,\emptyset) - d(X,Y)\right)/\left(d(X,\emptyset) + d(\bar{Y},\emptyset)\right)$, where $\bar{Y} = (\tilde{Y}\backslash Z^*) \cap Y$ denotes the set of tracks in Y that did not make it into Z^*, and $\emptyset$ contains the appropriate number of dummy tracks, being $|X|$ for $d(X,\emptyset)$ and $|\bar{Y}|$ for $d(\bar{Y},\emptyset)$. A track in Y may not have been selected for Z^* because either another estimated track or a dummy track was preferred over it. Such a spurious track typically consists of a combination of positions corresponding to different ground-truth tracks or to erroneous positions originating from clutter. The value of $\beta(X,Y)$ lies in $[0, \alpha(X,Y)]$. It takes value $\alpha(X,Y)$ if there are no spurious tracks in Y, that is if $\bar{Y} = \emptyset$. And it converges to 0 as the number of spurious tracks increases.

The measures α and β account for both association errors and localization errors. For a more detailed analysis, it is useful to separate these two types of errors. To evaluate association performance, the positions at time t of two paired tracks, $\theta_k^X(t)$ and $\theta_k^{Z^*}(t)$, are counted as matching if they are both non-dummy and $\left\|\theta_k^X(t) - \theta_k^{Z^*}(t)\right\|_2 < \epsilon$. Otherwise, they are counted as non-matching, with the exception of two dummy positions, which are not counted. This leads to the following measures:

3) The number of matching positions of the optimal pairs of tracks (X, Z^*). These are referred to as true positive (TP) position pairs.
4) The number of positions in X that are paired with a dummy position in Z^*. These are referred to as false negative (FN) position pairs because the dummy positions *are* nevertheless associated with track positions in the ground-truth set X.
5) The number of positions in the spurious tracks $\bar{Y}$ and the non-matching positions in Z^*. These are referred to as false positive (FP) positions because they correspond to estimated positions that were not associated with track positions in the ground-truth set X.
6) The Jaccard similarity coefficient[12] for positions, defined as $\mathrm{JSC} \triangleq \mathrm{TP}/(\mathrm{TP} + \mathrm{FN} + \mathrm{FP})$, which lies in the interval $[0,1]$. It takes value 1 only if all position pairs in (X, Z^*) are matching and $\bar{Y} = \emptyset$. It converges to 0 as the number of non-matching pairs and/or positions in $\bar{Y}$ increases.

It may also be useful to evaluate the association performance at the track level, rather than the position level. This leads to the following measures, analogous to the previous four measures:

7) The number of non-dummy tracks in Z^*. They are referred to as true positive tracks (TP_θ) because each of them contains a majority of matching positions with a single associated track in X.
8) The number of dummy tracks in Z^*. These are referred to as false negative tracks (FN_θ) because each of them *is* nevertheless associated with a single track in X.
9) The number of tracks in $\bar{Y}$. These are referred to as false positive tracks (FP_θ) because none of them is associated with a track in X.
10) The Jaccard similarity coefficient for tracks, defined as $\mathrm{JSC}_\theta \triangleq \mathrm{TP}_\theta/(\mathrm{TP}_\theta + \mathrm{FN}_\theta + \mathrm{FP}_\theta)$, which lies in the interval $[0,1]$. It takes value 1 only if Z^* does not contain dummy tracks and $\bar{Y} = \emptyset$. It converges to 0 as the number of dummy tracks in Z^* and/or tracks in $\bar{Y}$ increases.

Finally, the localization performance is characterized by the Euclidean distance (referred to as the error) between the positions of paired tracks. Since non-matching positions are already penalized by the above measures, the computation of localization errors is limited to matching positions:

11) The root mean-square error (RMSE) in true positive position pairs (TP as above).
12) The minimum error (Min) in TP position pairs.
13) The maximum error (Max) in TP position pairs.

14) The standard deviation (SD) of error in TP position pairs.

EXAMPLE CASES AND PERFORMANCE VALUES

To illustrate the effect of various tracking errors on the different performance measures, we provide several synthetic examples of increasing complexity. In the examples, a track is graphically represented as a series of point markers whose centers indicate the spatial position of the underlying particle at different time points, which are projected into a single image (**Figure N1**). The progression through time is indicated by a line connecting the point markers of the track. Tracks from the ground-truth set X are indicated by square-shaped markers connected by solid lines, while tracks from the estimated set Y are indicated by cross-shaped markers connected by dotted lines.

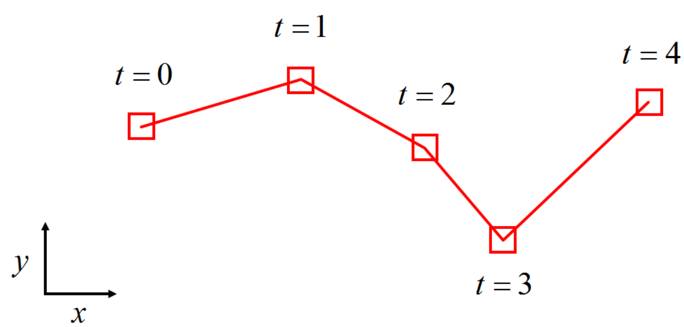

Figure N1: Ground-truth track defined for five successive time points ($t = 0, \dots, 4$). In the sequel we will omit the time labels from the point markers and consider the left-most marker as the starting point of the track.

CASE 1: NO ESTIMATED TRACKS

We start with the pathological case in which we have a single particle with ground truth (X as given in **Figure N1**) and the particle tracking method could not find any part of the track ($Y = \emptyset$). In this case, by definition, $\alpha = \beta = 0$. Also, since there are no estimated tracks at all, we have $\mathrm{TP}_\theta = \mathrm{FP}_\theta = 0$, and instead a dummy track is paired with the ground-truth track, yielding $\mathrm{FN}_\theta = 1$, leading to $\mathrm{JSC}_\theta = 0$. Similarly, we have $\mathrm{TP} = \mathrm{FP} = 0$, and since the ground-truth track covers five time points, we have the same number of matching dummy positions, $\mathrm{FN} = 5$, yielding $\mathrm{JSC} = 0$. Without any TP positions it is not possible to assess the localization performance of the tracking method.

CASE 2: ESTIMATED TRACKS IDENTICAL TO GROUND-TRUTH TRACKS

The other extreme is the case where the output of the particle tracking method is identical to the ground truth ($Y = X$ as in **Figure N2**). In this case, the number of matching tracks in Y is exactly the number of elements in X, here $\mathrm{TP}_\theta = 1$, and since there are no dummy or spurious tracks, we have $\mathrm{FN}_\theta = \mathrm{FP}_\theta = 0$, and thus $\mathrm{JSC}_\theta = 1$. Similarly, the number of positions in Y is exactly the number of positions in X, here $\mathrm{TP} = 5$, and $\mathrm{FN} = \mathrm{FP} = 0$, yielding $\mathrm{JSC} = 1$. Because the distance between each pair of estimated and ground-truth positions is 0, we have $\alpha = \beta = 1$, and all localization errors are 0.

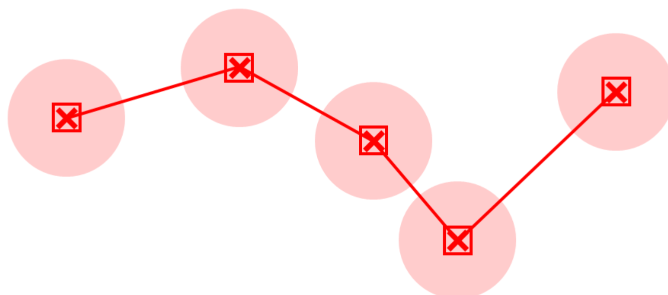

Figure N2: Ground-truth track (square-shaped markers connected by a solid line) with a perfectly matching estimated track (cross-shaped markers connected by a dotted line that is fully overlapping and thus not visible). The larger, light colored circles around the ground-truth positions indicate the gate ϵ within which estimated positions are searched.

CASE 3: FULLY MATCHING BUT NOT IDENTICAL TRACKS

In this example, we consider a similar situation as in Case 2, where an optimal pairing of estimated and ground-truth tracks is possible without the need for dummy tracks and without leaving spurious tracks, but where the estimated positions are not identical to the ground-truth positions, although they are within the gates of the latter (**Figure N3**). The distortions in the estimated positions affect only α and β and the localization measures. Since there are no spurious tracks, $\beta = \alpha$, and in this example both drop to 0.364. The localization errors all become > 0 (we refer to **Table N1** at the end of this section for the values of the performance measures for all example cases discussed).

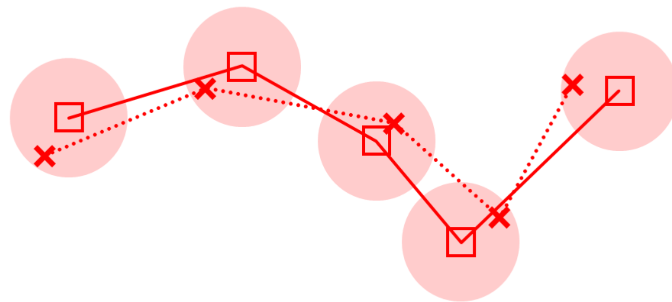

Figure N3: Ground-truth track (squares connected by solid lines) with a paired estimated track (crosses connected by dotted lines) whose positions are not identical with, but fall within, the gates (light colored circles) of the ground-truth positions.

CASE 4: MATCHING TRACKS HAVING NON-MATCHING POSITIONS

Next, we reconsider the situation of Case 3, and move one of the estimated track positions out of the gate of the corresponding ground-truth track position (**Figure N4**).

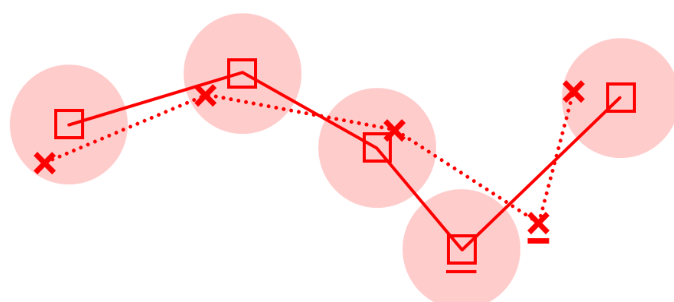

Figure N4: Estimated track (crosses connected by dotted lines) that is paired with a ground-truth track (squares connected by solid lines) but with one position (underscored cross) falling outside the gate of the ground-truth track position (underscored square).

As a result, this estimated position is considered non-matching with the ground truth, which translates into an increase in the number of false-positive positions, $\mathrm{FP} = 1$. At the same time, the ground-truth position is now matched with a dummy position, leading to $\mathrm{FN} = 1$, and since $\mathrm{TP} = 4$, we have $\mathrm{JSC} = 0.667$. The dummy position receives a penalty that is larger than the localization error of the original position in Case 3, leading to a decrease of both α and β. Also, the computation of RMSE, which is limited to TP positions only, no longer includes the now non-matching position, whose localization error was relatively large, and as a result RMSE slightly decreases while SD slightly increases. All other performance measures remain unaffected by the change (**Table N1**).

CASE 5: MATCHING TRACKS WITH BIRTH AND DEATH MISMATCHES

In the previous examples we compared tracks (estimated versus ground truth) that were defined over the same time interval. Now we consider a case in which the existence window of the estimated track is shifted by one time point compared to the ground-truth track (**Figure N5**). This is representative of cases where a particle tracking method fails to detect the right birth and death times of a particle.

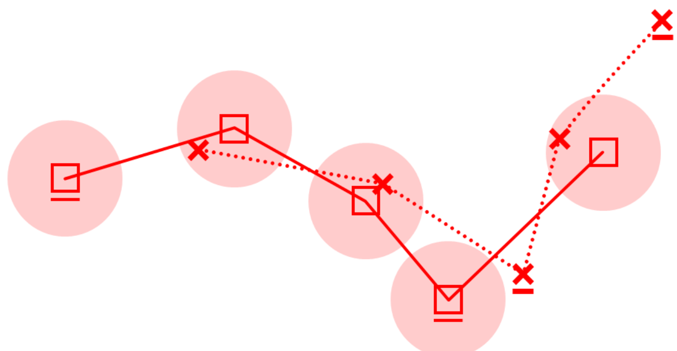

Figure N5: Estimated track (crosses connected by dotted lines) that is paired with a ground-truth track (squares connected by solid lines) but whose start and end positions do not correspond to the ground truth (all problematic positions are underscored).

Since the majority of the track positions still match, the two tracks are paired by the track association algorithm, and similar to Case 4 only the performance measures accounting for position matching and localization errors are affected (**Table N1**). Specifically, since TP further decreases to 3, and FP increases to 2, and the number of matched dummy positions FN also increases to 2, JSC drops to 0.429, and both α and β significantly drop to 0.052. Because we are now missing one more position (the left-most) with a relatively large localization error, again RMSE slightly decreases and SD slightly increases.

CASE 6: MULTIPLE ESTIMATED AND GROUND-TRUTH TRACKS

In the following examples (Cases 6-10) we consider multiple estimated and ground-truth tracks. Together they cover all the types of situations encountered in our study. As a first example we extend Case 5 with one additional estimated and corresponding ground-truth track (**Figure N6**).

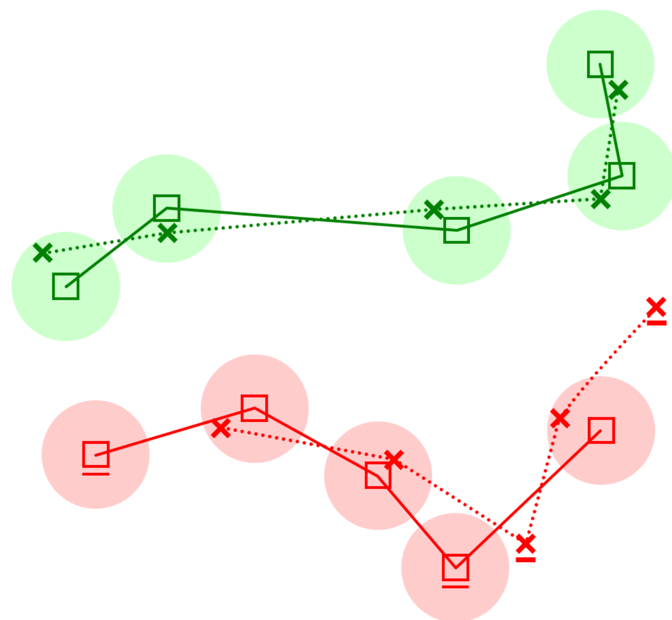

Figure N6: Multiple estimated tracks (crosses connected by dotted lines) that are paired with corresponding ground-truth tracks (squares connected by solid lines). The pairing produced by the track association algorithm is indicated by the coloring (the red estimated track is paired with the red ground-truth track, and similar for the green tracks).

Since the newly added estimated track and ground-truth track match both spatially and temporally, with only small inaccuracies in the estimated positions, they are correctly paired by the track association algorithm. As a result, similar to Case 5, only the performance measures accounting for position matching and localization errors are affected (**Table N1**). In particular, TP now increases to 8, making JSC increase again to 0.667, and also α and β increase significantly, to 0.256. Because the newly added estimated track has relatively low localization errors, both RMSE and SD decrease.

CASE 7: MISSING ESTIMATED TRACKS

In this example we revisit Case 6 and consider the situation in which there are two ground-truth tracks but only one estimated track (**Figure N7**). This is representative of cases where a particle tracking method produces less tracks than the number of ground-truth tracks. The estimated track is paired with one of the ground-truth tracks (the one whose positions match best) while the remaining ground-truth track is matched with a dummy track. As a result, $\mathrm{TP}_\theta = \mathrm{FN}_\theta = 1$, and since still $\mathrm{FP}_\theta = 0$, we have $\mathrm{JSC}_\theta = 0.5$. Because the number of TP positions decreases back to the level of Case 5, while the number of FN positions increases by the same amount, the value of JSC now drops significantly, as do the values of both α and β (**Table N1**). And since localization errors are computed only for TP positions, the values of the corresponding performance measures are the same as in Case 5.

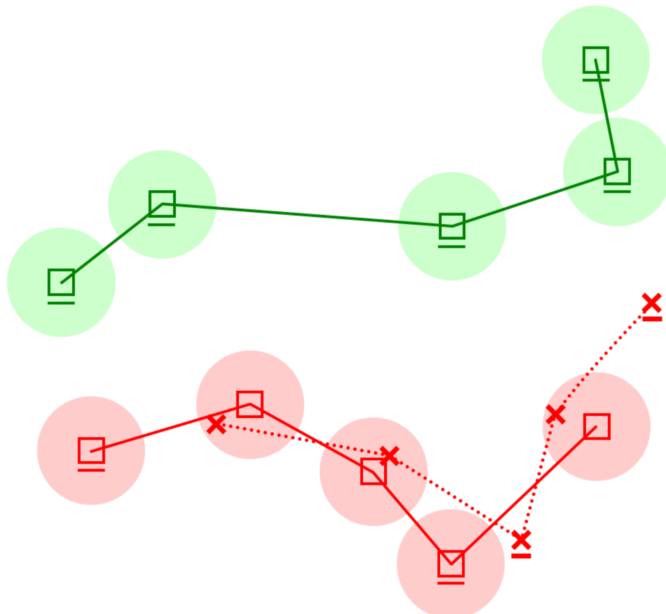

Figure N7: Two ground-truth tracks (squares connected by solid lines) but only one estimated track (crosses connected by dotted lines). The pairing produced by the track association algorithm is indicated by the coloring (the red estimated track is paired with the red ground-truth track while the green ground-truth track is paired with a dummy).

CASE 8: SPURIOUS ESTIMATED TRACKS

Here we again revisit Case 6, but consider the reverse situation as in Case 7, in that we now have two estimated tracks but only one ground-truth track (**Figure N8**). This is representative of cases where a particle tracking method produces more tracks than the number of ground-truth tracks. One of the estimated tracks (whose positions match best) is paired with the single ground-truth track while the other estimated track is considered spurious and remains non-paired. As a result, $\mathrm{TP}_\theta = \mathrm{FP}_\theta = 1$, and since $\mathrm{FN}_\theta = 0$, we still have $\mathrm{JSC}_\theta = 0.5$. Because the FN and FP positions are interchanged compared to Case 7, the value of JSC remains the same, but α increases back to the level of Case 5 (**Table N1**), as it does not penalize spurious tracks. And since localization errors are computed only for TP positions, the values of the corresponding performance measures are the same as in Cases 5 and 7.

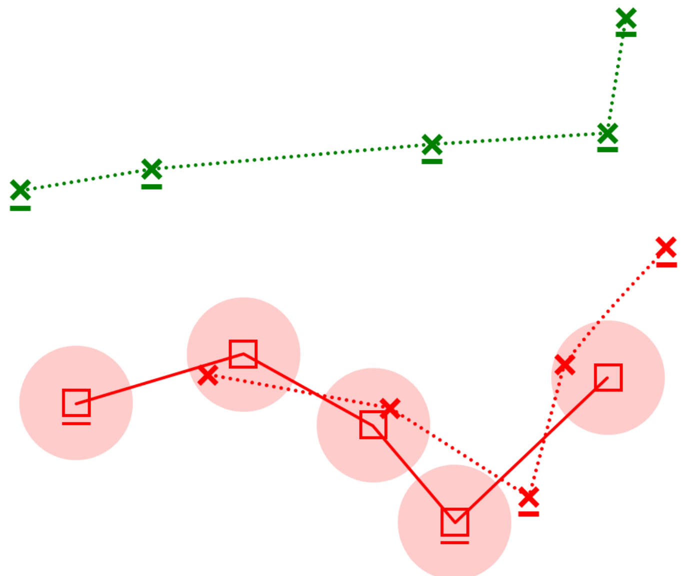

Figure N8: Two estimated tracks (crosses connected by dotted lines) but only one ground-truth track (squares connected by solid lines). The pairing produced by the track association algorithm is indicated by the coloring (the red estimated track is paired with the single ground-truth track while the green estimated track is considered spurious).

CASE 9: BROKEN ESTIMATED TRACKS

In this example we consider a similar situation as in Case 6 but with one of the two estimated tracks being broken into two pieces (**Figure N9**). This is representative of cases where a particle tracking method fails to bridge relatively large gaps between successive positions and produces broken tracks. In this case only one of the broken track segments (the one that matches best) is paired with the corresponding ground-truth track, and the other is considered spurious, yielding $\mathrm{FP}_\theta = 1$. Since all ground-truth tracks are paired, we have $\mathrm{TP}_\theta = 2$ and $\mathrm{FN}_\theta = 0$, resulting in $\mathrm{JSC}_\theta = 0.667$. Similarly, for the track positions we find $\mathrm{FP} = 4$ (two spurious and two non-matching positions), $\mathrm{FN} = 4$ (ground-truth positions paired with a dummy position), and $\mathrm{TP} = 6$, resulting in $\mathrm{JSC} = 0.429$. Compared to Cases 7 and 8, both α and β increase because of a better matching of points, but they remain lower than in Case 6, and due to the presence of a spurious track they continue to have different values (**Table N1**). Since localization errors are computed only for TP positions, the values of the corresponding performance measures are similar to those in Case 6.

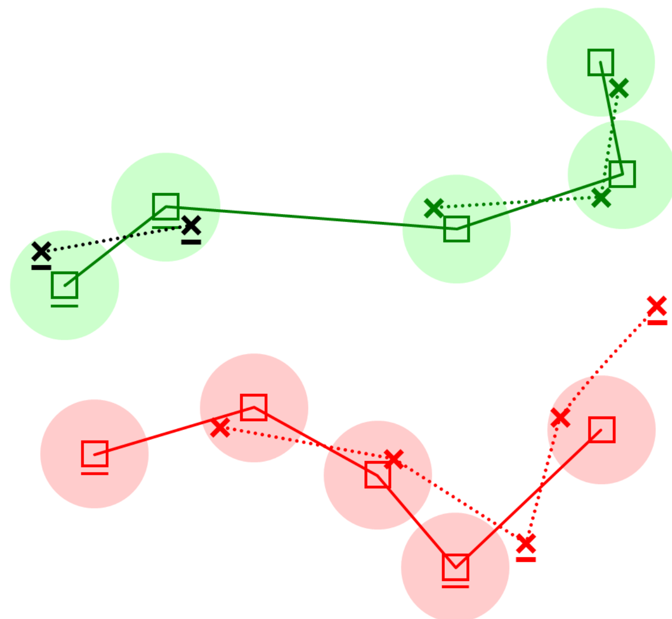

Figure N9: Two ground-truth tracks (squares connected by solid lines) but three estimated tracks (crosses connected by dotted lines) resulting from a linking failure. The pairing produced by the track association algorithm is indicated by the coloring (the red estimated track is paired with the red ground-truth track, and the green estimated track is paired with the green ground-truth track, while the black estimated track is considered spurious).

CASE 10: MIXED UP ESTIMATED TRACKS

In this last example we revisit Case 9 and consider estimated tracks consisting of detected particle positions belonging to different ground-truth tracks (**Figure N10**). This is representative of cases where a particle tracking method erroneously switches particle tracks. In the particular case considered here, one estimated track is paired with one of the ground-truth tracks, thus $\mathrm{TP}_\theta = 1$, but the other estimated track does not match with the other ground-truth track, as the majority of its positions is too far off. It thus remains non-paired, yielding $\mathrm{FP}_\theta = 1$, and consequently the ground-truth track is paired with a dummy track, yielding $\mathrm{FN}_\theta = 1$, and thus $\mathrm{JSC}_\theta = 0.333$. In terms of positions we find that $\mathrm{FP} = \mathrm{FN} = 7$, while $\mathrm{TP} = 3$, resulting in $\mathrm{JSC} = 0.176$. Both α and β decrease compared to Case 9 (**Table N1**) while remaining different from each other due to the presence of a spurious track. Finally, the localization errors are now computed based on only three TP positions, and are relatively small.

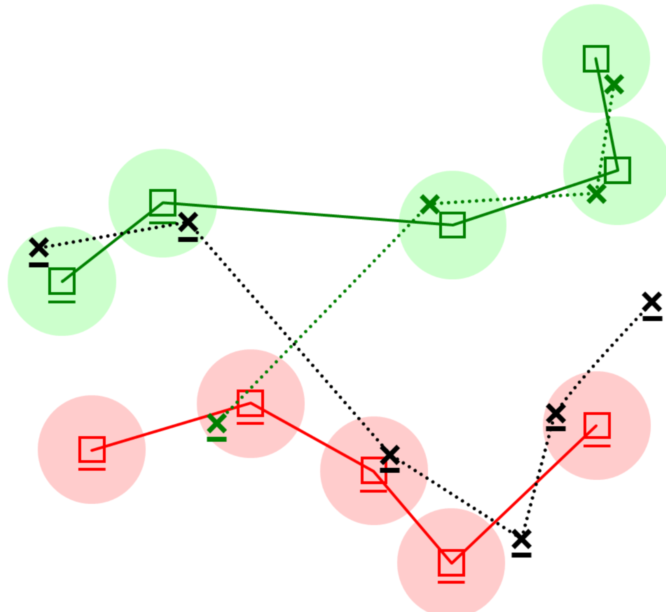

Figure N10: Two ground-truth tracks (squares connected by solid lines) and two estimated tracks (crosses connected by dotted lines) resulting from linking failures. The pairing produced by the track association algorithm is indicated by the coloring (the green estimated track is paired with the green ground-truth track while the black estimated track is not paired but considered spurious).

Table N1: Overview of the performance values for all discussed example cases. All floating-point values are given with three decimal places.

Case	α	β	TP	FN	FP	JSC	TP_θ	FN_θ	FP_θ	JSC_θ	RMSE	Min	Max	SD
1	0.000	0.000	0	5	0	0.000	0	1	0	0.000	-	-	-	-
2	1.000	1.000	5	0	0	1.000	1	0	0	1.000	0.000	0.000	0.000	0.000
3	0.364	0.364	5	0	0	1.000	1	0	0	1.000	3.317	1.414	4.123	0.935
4	0.308	0.308	4	1	1	0.667	1	0	0	1.000	3.240	1.414	4.123	1.018
5	0.052	0.052	3	2	2	0.429	1	0	0	1.000	3.109	1.414	4.123	1.121
6	0.256	0.256	8	2	2	0.667	2	0	0	1.000	2.894	1.414	4.123	0.822
7	0.026	0.026	3	7	2	0.250	1	1	0	0.500	3.109	1.414	4.123	1.121
8	0.052	0.026	3	2	7	0.250	1	0	1	0.500	3.109	1.414	4.123	1.121
9	0.168	0.140	6	4	4	0.429	2	0	1	0.667	2.887	1.414	4.123	0.828
10	0.142	0.089	3	7	7	0.176	1	1	1	0.333	2.646	2.236	2.828	0.279

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